# Finding Limits Algebraically - Classwork

We are going to now determine limits without benefit of looking at a graph, that is $\lim_{x \to a} f(x)$.

There are three steps to remember:
1) plug in $a$
2) Factor/cancel and go back to step 1
3) $\infty$, $-\infty$, or DNE

Example 1) find $\lim_{x \to -2} x^2 - 4x + 1$

You can do this by plugging in.

Example 2) find $\lim_{x \to -2} \frac{2x-6}{x-2}$

You can also do this by plugging in.

Example 3) find $\lim_{x \to -2} \frac{x^2 - 2x - 8}{x^2 - 4}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

Example 4) find $\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^3 - 1}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

If steps 1 and 2 do not work (you have a zero in the denominator, your answer is one of the following:

| $\infty$ | $-\infty$ | **Does Not Exist (DNE)** |
|---|---|---|

To determine which, you must split your limit into two separate limits.: $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a+} f(x)$. Make a sign chart by plugging in a number close to $a$ on the left side and determining its sign. You will also plug in a number close to $a$ on the right side and determine its sign. **Each of these will be some form of $\infty$, either positive or negative.** Only if they are the same will the limit be $\infty$ or $-\infty$.

What this says is that in this case, $\lim_{x \to a^-} f(x) =$ some form of $\infty$ and $\lim_{x \to a+} f(x) =$ some form of $\infty$

You need to check whether they are the same.

Example 5) find $\lim_{x \to 2} \frac{2x+5}{x-2}$

Step 1) Plug in - $\frac{9}{0}$ - no good  Step 2) - No factoring/cancel  So your answer is $\infty$, $-\infty$ or DNE

Example 6) find $\lim_{x \to 0} \frac{4}{x^2}$

Step 1) Plug in - $\frac{4}{0}$ - no good Step 2) - No factoring/cancel So your answer is $\infty$, $-\infty$ or DNE

Example 7) find $\lim_{x \to -3} \frac{x^2+2x-3}{x^2+6x+9}$

Example 8) find $\lim_{x \to 2} \frac{2x-4}{x^3-6x^2+12x-8}$

Example 9) $f(x) = \begin{cases} x^2-4, x \geq 1 \\ -2x-1, x < 1 \end{cases}$ find $\lim_{x \to 1} f(x)$

Example 10) $f(x) = \begin{cases} \frac{x}{x-2}, x \geq 2 \\ \frac{x-3}{x-2}, x < 2 \end{cases}$ find $\lim_{x \to 2} f(x)$

Example 11) $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

Finally, we are interested also in problems of the type: $\lim_{x \to \pm\infty} f(x)$. Here are the rules:

Write $f(x)$ as a fraction.

1) If the highest power of $x$ appears in the denominator (bottom heavy), $\lim_{x \to \pm\infty} f(x) = 0$
2) If the highest power of $x$ appears in the numerator (top heavy), $\lim_{x \to \pm\infty} f(x) = \pm\infty$ plug in very large or small numbers and determine the sign of the answer
3) If the highest power of $x$ appears both in the numerator and denominator (powers equal), $\lim_{x \to \pm\infty} f(x) = \frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$

Example 12) $\lim_{x \to \infty} \frac{4x^2+50}{x^3-85}$

Example 13) $\lim_{x \to \infty} \frac{4x^3-5x^2+3x-1}{5x^3-7x-25}$

Example 14) $\lim_{x \to \infty} \frac{3x^3-23}{4x-1}$

Example 15) $\lim_{x \to -\infty} \frac{4x-5x^2+3}{\frac{1}{x}}$

Example 16) $\lim_{x \to \infty} \frac{\sqrt{x^2-3x}}{2x+1}$

Example 17) $\lim_{x \to -\infty} \frac{\sqrt{x^2-3x}}{2x+1}$