Algebra 1
Investigating Patterns: Exponent Properties

Here are some exponential expressions involving very large quantities:

a. $2^{1000} \cdot 2^3$

b. $(3^{2000})^4$

c. $\frac{5^{1000}}{5^{100}}$

Even though they are too big for your calculator to handle, they can be simplified.

Investigate patterns to understand

- multiplication of exponential expressions with like bases
- exponential quantities raised to an exponent
- division of exponential expressions with like bases

Calculate:

$2^1 =$	$2^5 =$	$2^9 =$
$2^2 =$	$2^6 =$	$2^{10} =$
$2^3 =$	$2^7 =$	$2^{11} =$
$2^4 =$	$2^8 =$	$2^{12} =$

Can you find out how to do problems of the form of a,b,and c above?
Show examples, and explain your method.

Write mathematical rules for your conclusions. Verify and justify your rules. Use other paper if needed.

Use your rules to answer questions a, b, and c.

Criterion B: Investigating patterns

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: 1. apply, with teacher support, mathematical problem-solving techniques to discover simple patterns 2. state predictions consistent with patterns.
3–4	The student is able to: 1. apply mathematical problem-solving techniques to discover simple patterns 2. suggest general rules consistent with findings for patterns.
5–6	The student is able to: 1. select and apply mathematical problem-solving techniques to discover complex patterns 2. describe patterns as general rules consistent with findings 3. verify the validity of these general rules.
7–8	The student is able to: 1. select and apply mathematical problem-solving techniques to discover complex patterns 2. describe patterns as general rules consistent with correct findings 3. prove, or verify and justify, these general rules.

Lesson 6.3 • Multiplication and Exponents

Name _____ Period _____ Date _____

1. Use the properties of exponents to rewrite each expression. Use your calculator to check that your expression is equivalent to the original expression.

a. $(-7)(w)(w)(w)$

b. $(3)(a)(a)(a)(b)(b)(b)(b)$

c. $(5)(p)(p)(p)(-3)(q)(q)$

d. $4x^2 \cdot 3x^4$

e. $(6c)(-2c^3)(3d^2)$

f. $(-4m^3)(2m + m^2)$

2. Write each expression in expanded form. Then rewrite the product in exponential form.

a. $4^3 \cdot 4^4$

b. $(-3)^5 \cdot (-3)^2$

c. $(-2)^8(-2)^7$

d. $(8^6)(8^3)$

e. $x^9 \cdot x^4$

f. $n \cdot n^9$

3. Rewrite each expression with a single exponent.

a. $(4^5)^5$

b. $(8^2)^7$

c. $(x^9)^4$

d. $(y^3)^{10}$

e. $(5^3)^7$

f. $[(-3)^3]^2$

g. $(z^8)^2$

h. $(10^9)^3$

i. $(0.5^2)^5$

j. $(100^3)^8$

k. $[(-6)^5]^4$

l. $(t^7)^2$

4. Use the properties of exponents to rewrite each expression.

a. $4x \cdot 3x$

b. $(6m)(2m^2)$

c. $(-5n^2)(4n^4)$

d. $xy^2 \cdot x^2y^4$

e. $(2x^4)^6$

f. $(-4m^5)^2$

g. $(-3m^4n^7)^3$

h. $(5x^2yz^5)^4$

i. $(-3x^4y^3)^3$

5. Evaluate each expression for the given value of the variables.

a. $2x^3$ for $x = -5$

b. $5y^4$ for $y = -3$

c. $x^2 - 3x + 2$ for $x = 4$

d. $-5x^3y^2$ for $x = -2$ and $y = -1$

6. Match expressions from this list that are equivalent but written in different forms. There can be multiple matches.

a. $(2x^2)^3$

b. $8x^5$

c. $(-4x^3)(-2x^3)$

d. $(6x^2)(2x^3)$

e. $(12)(x)(x)(x)(x)(x)$

f. $(4x)(2x^5)$

Lesson 6.5 • Looking Back with Exponents

Name _____ Period _____ Date _____

1. Eliminate factors equivalent to 1 and rewrite the right side of this equation.

$$\frac{p^3q^5r^2}{pq^3r^2} = \frac{p \cdot p \cdot p \cdot q \cdot q \cdot q \cdot q \cdot r \cdot r}{p \cdot q \cdot q \cdot q \cdot r \cdot r}$$

2. Use the properties of exponents to rewrite each expression.

a. $\frac{m^{10}}{m^4}$ b. $\frac{n^8}{n}$ c. $\frac{24x^9}{8x^5}$

d. $\frac{36x^5y^6}{4xy^3}$ e. $\frac{45m^7n^4}{-9m^4n^2}$ f. $\frac{-50x^{12}y^8}{-2x^{11}y^6}$

g. $\frac{42x^{10}y^5}{6x^3y}$ h. $\frac{-12m^5n^7}{-3m^4n^2}$ i. $\frac{-15r^{12}s^5}{5r^4s^2}$

3. Lana bought a car 8 years ago. Since she purchased it, the value of the car has decreased by 12% each year. The car is now worth about $5900.

a. Which letter in the equation $y = A(1 - r)^x$ could represent the value of the car 8 years ago when Lana bought it?

b. Substitute the other given information into the equation $y = A(1 - r)^x$.

c. Solve your equation in 3b to find the value of Lana's car when she bought it.

4. Use the properties of exponents to rewrite each expression.

a. $(-3x)^2(2x^2)^4$ b. $\frac{(-4y^2)^6}{(-4y^2)^5}$

c. $\frac{(4z^2)^3}{(2z)^2}$ d. $(3a^2b)^2(-2ab)^3$

e. $\frac{4.2 \times 10^9}{1.2 \times 10^5}$ f. $\frac{(5r^3s^6)(4rs^2)^2}{20r^4s^8}$

5. a. In 2004 Canada had a population of about 3.25×10^7 people. Canada has an area of approximately 3.51×10^6 square miles. Find the population density of Canada (the number of people per square mile).

b. In 2004 the United States had a population of about 2.93×10^8 people. The United States has an area of approximately 3.54×10^6 square miles. Find the population density of the United States.

c. How did the population densities of Canada and the United States in 2004 compare?

(*The World Almanac and Book of Facts 2005*, p. 848)