77. $y = \dfrac{2}{e^x + e^{-x}}$

78. $y = \dfrac{e^x - e^{-x}}{2}$

79. $y = x^2e^x - 2xe^x + 2e^x$

80. $y = xe^x - e^x$

81. $f(x) = e^{-x}\ln x$

82. $f(x) = e^3 \ln x$

83. $y = e^x(\sin x + \cos x)$

84. $y = \ln e^x$

85. $g(x) = \ln x^2$

86. $h(x) = \ln(2x^2 + 3)$

87. $y = (\ln x)^4$

88. $y = x \ln x$

89. $y = \ln\left(x\sqrt{x^2 - 1}\right)$

90. $y = \ln\sqrt{x^2 - 9}$

91. $f(x) = \ln\left(\dfrac{x}{x^2 + 1}\right)$

92. $f(x) = \ln\left(\dfrac{2x}{x + 3}\right)$

93. $g(t) = \dfrac{\ln t}{t^2}$

94. $h(t) = \dfrac{\ln t}{t}$

95. $y = \ln\sqrt{\dfrac{x + 1}{x - 1}}$

96. $y = \ln\sqrt[3]{\dfrac{x - 2}{x + 2}}$

97. $y = \dfrac{-\sqrt{x^2 + 1}}{x} + \ln\left(x + \sqrt{x^2 + 1}\right)$

98. $y = \dfrac{-\sqrt{x^2 + 4}}{2x^2} - \dfrac{1}{4}\ln\left(\dfrac{2 + \sqrt{x^2 + 4}}{x}\right)$

99. $y = \ln|\sin x|$

100. $y = \ln|\csc x|$

101. $y = \ln\left|\dfrac{\cos x}{\cos x - 1}\right|$

102. $y = \ln|\sec x + \tan x|$

103. $y = \ln\left|\dfrac{-1 + \sin x}{2 + \sin x}\right|$

104. $y = \ln\sqrt{1 + \sin^2 x}$

Find the Error In Exercises 105–108, describe and correct the error in finding the derivative of the function.

105. If $y = (1 - x)^{1/2}$, then ~~$y' = \frac{1}{2}(1 - x)^{-1/2}$~~.

106. If $f(x) = \sin^2 2x$, then ~~$f'(x) = 2(\sin 2x)(\cos 2x)$~~.

107. If $y = \dfrac{4^{3x}}{x}$, then

~~$y' = \dfrac{x(\ln 4)4^{3x} - 4^{3x}}{x^2} = \dfrac{4^{3x}(x \ln 4 - 1)}{x^2}$~~.

108. If $g(x) = x^4e^{-2x}$, then

~~$g'(x) = x^4e^{-2x} + e^{-2x}(4x^3) = x^3e^{-2x}(x + 4)$~~.

Evaluating a Derivative In Exercises 109–116, find and evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.

109. $y = \sqrt{x^2 + 8x}$, $(1, 3)$

110. $y = \sqrt[5]{3x^3 + 4x}$, $(2, 2)$

111. $f(x) = \dfrac{5}{x^3 - 2}$, $\left(-2, -\dfrac{1}{2}\right)$

112. $f(x) = \dfrac{1}{(x^2 - 3x)^2}$, $\left(4, \dfrac{1}{16}\right)$

113. $f(t) = \dfrac{3t + 2}{t - 1}$, $(0, -2)$

114. $f(x) = \dfrac{x + 4}{2x - 5}$, $(9, 1)$

115. $y = 26 - \sec^3 4x$, $(0, 25)$

116. $y = \dfrac{1}{x} + \sqrt{\cos x}$, $\left(\dfrac{\pi}{2}, \dfrac{2}{\pi}\right)$

Finding an Equation of a Tangent Line In Exercises 117–124, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *tangent* feature of the graphing utility to confirm your results.

117. $f(x) = \sqrt{2x^2 - 7}$, $(4, 5)$

118. $f(x) = (9 - x^2)^{2/3}$, $(1, 4)$

119. $f(x) = \sin 8x$, $(\pi, 0)$

120. $y = \cos 3x$, $\left(\dfrac{\pi}{4}, -\dfrac{\sqrt{2}}{2}\right)$

121. $f(x) = \tan^2 x$, $\left(\dfrac{\pi}{4}, 1\right)$

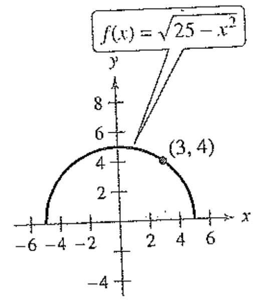
122. $y = 2\tan^3 x$, $\left(\dfrac{\pi}{4}, 2\right)$

123. $y = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right)$, $(0, 4)$

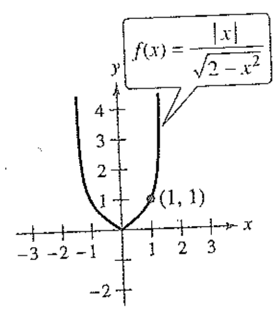
124. $y = 2e^{1-x^2}$, $(1, 2)$

Famous Curves In Exercises 125 and 126, find an equation of the tangent line to the graph at the given point. Then use a graphing utility to graph the function and its tangent line in the same viewing window.

125. Top half of circle



126. Bullet-nose curve



127. **Horizontal Tangent Line** Determine the point(s) in the interval $(0, 2\pi)$ at which the graph of $f(x) = 2\cos x + \sin 2x$ has a horizontal tangent.

128. **Horizontal Tangent Line** Determine the point(s) at which the graph of

$$f(x) = \frac{x}{\sqrt{2x - 1}}$$

has a horizontal tangent.

Finding a Second Derivative In Exercises 129–136, find the second derivative of the function.

129. $f(x) = 5(2 - 7x)^4$

130. $f(x) = 6(x^3 + 4)^3$

131. $f(x) = \dfrac{1}{x - 6}$

132. $f(x) = \dfrac{8}{(x - 2)^2}$

133. $f(x) = \sin x^2$

134. $f(x) = \sec^2 \pi x$

135. $f(x) = (3 + 2x)e^{-3x}$

136. $g(x) = \sqrt{x} + e^x \ln x$

Evaluating a Second Derivative In Exercises 137–140, evaluate the second derivative of the function at the given point. Use a computer algebra system to verify your result.

137. $h(x) = \frac{1}{9}(3x + 1)^3, \left(1, \frac{64}{9}\right)$

138. $f(x) = \frac{1}{\sqrt{x + 4}}, \left(0, \frac{1}{2}\right)$

139. $f(x) = \cos x^2, (0, 1)$ **140.** $g(t) = \tan 2t, \left(\frac{\pi}{6}, \sqrt{3}\right)$

Finding a Derivative In Exercises 141–154, find the derivative of the function.

141. $f(x) = 3^x$ **142.** $g(x) = 5^{-x}$

143. $y = 4^{2x-3}$ **144.** $y = x(6^{-2x})$

145. $g(t) = t^2 2^t$ **146.** $f(t) = \frac{3^{2t}}{t}$

147. $h(\theta) = 2^{-\theta} \cos \pi\theta$ **148.** $g(\alpha) = 5^{-\alpha/2} \sin 2\alpha$

149. $y = \log_3 x$ **150.** $h(x) = \log_3 \frac{x\sqrt{x - 1}}{2}$

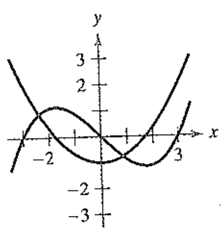
151. $y = \log_5 \sqrt{x^2 - 1}$ **152.** $y = \log_{10} \frac{x^2 - 1}{x}$

153. $g(t) = \frac{10 \log_4 t}{t}$ **154.** $f(t) = t^{3/2} \log_2 \sqrt{t + 1}$

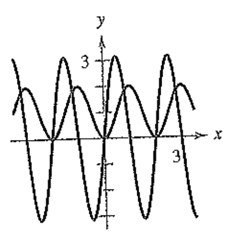
WRITING ABOUT CONCEPTS

Identifying Graphs In Exercises 155 and 156, the graphs of a function f and its derivative f' are shown. Label the graphs as f or f' and write a short paragraph stating the criteria you used in making your selection. To print an enlarged copy of the graph, go to *MathGraphs.com*.

155.



156.



Describing a Relationship In Exercises 157 and 158, the relationship between f and g is given. Explain the relationship between f' and g'.

157. $g(x) = f(3x)$ **158.** $g(x) = f(x^2)$

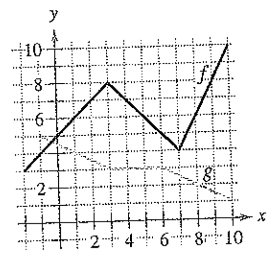
Using Relationships In Exercises 159–162, given that $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$, find $f'(5)$, if possible. If it is not possible, state what additional information is required.

159. $f(x) = g(x)h(x)$ **160.** $f(x) = g(h(x))$

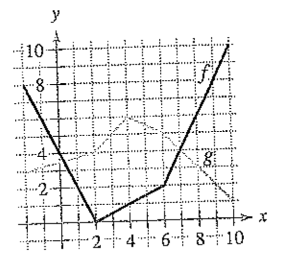
161. $f(x) = \frac{g(x)}{h(x)}$ **162.** $f(x) = [g(x)]^3$

Finding Derivatives In Exercises 163 and 164, the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.

163. (a) Find $h'(1)$.
(b) Find $s'(5)$.



164. (a) Find $h'(3)$.
(b) Find $s'(9)$.



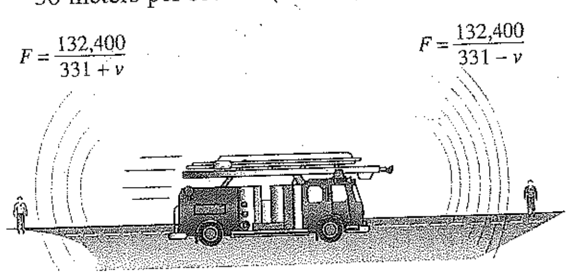
165. Doppler Effect The frequency F of a fire truck siren heard by a stationary observer is

$$F = \frac{132{,}400}{331 \pm v}$$

where $\pm v$ represents the velocity of the accelerating fire truck in meters per second (see figure). Find the rate of change of F with respect to v when

(a) the fire truck is approaching at a velocity of 30 meters per second (use $-v$).

(b) the fire truck is moving away at a velocity of 30 meters per second (use $+v$).



166. Harmonic Motion The displacement from equilibrium of an object in harmonic motion on the end of a spring is

$$y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$$

where y is measured in feet and t is the time in seconds. Determine the position and velocity of the object when $t = \pi/8$.

167. Pendulum A 15-centimeter pendulum moves according to the equation $\theta = 0.2 \cos 8t$, where θ is the angular displacement from the vertical in radians and t is the time in seconds. Determine the maximum angular displacement and the rate of change of θ when $t = 3$ seconds.

168. **Wave Motion** A buoy oscillates in simple harmonic motion $y = A \cos \omega t$ as waves move past it. The buoy moves a total of 3.5 feet (vertically) from its low point to its high point. It returns to its high point every 10 seconds.

(a) Write an equation describing the motion of the buoy if it is at its high point at $t = 0$.

(b) Determine the velocity of the buoy as a function of t.

169. **Modeling Data** The table shows the temperatures T (°F) at which water boils at selected pressures p (pounds per square inch). *(Source: Standard Handbook of Mechanical Engineers)*

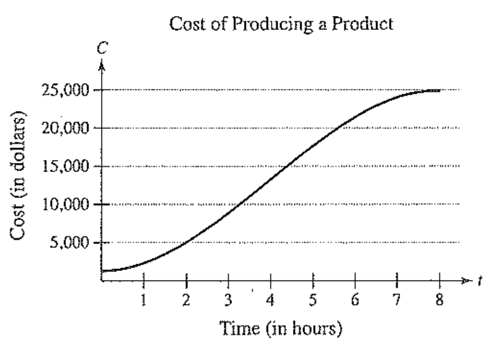
p	5	10	14.696 (1 atm)	20
T	162.24°	193.21°	212.00°	227.96°

p	30	40	60	80	100
T	250.33°	267.25°	292.71°	312.03°	327.81°

(a) Use the information in the table to approximate $T'(35)$ and $T'(70)$. Interpret your results using correct units.

(b) A model that approximates the data is $T = 87.97 + 34.96 \ln p + 7.91\sqrt{p}$. Use the model to find the rates of change of T with respect to p when $p = 35$ and $p = 70$. Compare the rates of change with the approximations in part (a).

170. **HOW DO YOU SEE IT?** The cost C (in dollars) of producing x units of a product is $C = 60x + 1350$. For one week, management determined that the number of units produced x at the end of t hours can be modeled by $x = -1.6t^3 + 19t^2 - 0.5t - 1$. The graph shows the cost C in terms of the time t.



(a) Using the graph, which is greater, the rate of change of the cost after 1 hour or the rate of change of the cost after 4 hours?

(b) Explain why the cost function is not increasing at a constant rate during the 8-hour shift.

171. **Think About It** The table shows some values of the derivative of an unknown function f. Complete the table by finding the derivative of each transformation of f, if possible.

(a) $g(x) = f(x) - 2$

(b) $h(x) = 2f(x)$

(c) $r(x) = f(-3x)$

(d) $s(x) = f(x + 2)$

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$						
$h'(x)$						
$r'(x)$						
$s'(x)$						

172. **Inflation** If the annual rate of inflation averages 5% over the next 10 years, the approximate cost c of goods or services during any year in that decade is $C(t) = P(1.05)^t$, where t is the time in years and P is the present cost.

(a) If the price of an oil change for your car is presently \$29.95, estimate the price 10 years from now.

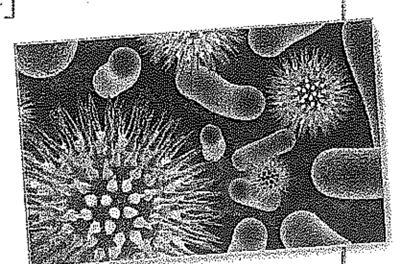
(b) Find the rates of change of C with respect to t when $t = 1$ and $t = 8$.

(c) Verify that the rate of change of C is proportional to C. What is the constant of proportionality?

173. **BIOLOGY**

The number N of bacteria in a culture after t days is modeled by

$$N = 400\left[1 - \frac{3}{(t^2 + 2)^2}\right].$$



Find the rate of change of N with respect to t when (a) $t = 0$, (b) $t = 1$, (c) $t = 2$, (d) $t = 3$, and (e) $t = 4$. (f) What can you conclude?

174. **Depreciation** The value V of a machine t years after it is purchased is inversely proportional to the square root of $t + 1$. The initial value of the machine is \$10,000.

(a) Write V as a function of t.

(b) Find the rate of depreciation when $t = 1$.

(c) Find the rate of depreciation when $t = 3$.

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175. Finding a Pattern Consider the function $f(x) = \sin \beta x$, where β is a constant.

(a) Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.

(b) Verify that the function and its second derivative satisfy the equation $f''(x) + \beta^2 f(x) = 0$.

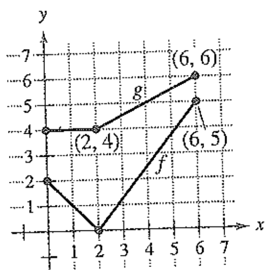
(c) Use the results of part (a) to write general rules for the even- and odd-order derivatives $f^{(2k)}(x)$ and $f^{(2k-1)}(x)$. [*Hint:* $(-1)^k$ is positive if k is even and negative if k is odd.]

176. Conjecture Let f be a differentiable function of period p.

(a) Is the function f' periodic? Verify your answer.

(b) Consider the function $g(x) = f(2x)$. Is the function g' periodic? Verify your answer.

177. Think About It Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$, where f and g are shown in the figure. Find (a) $r'(1)$ and (b) $s'(4)$.



178. Using Trigonometric Functions

(a) Find the derivative of the function $g(x) = \sin^2 x + \cos^2 x$ in two ways.

(b) For $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$, show that $f'(x) = g'(x)$.

179. Even and Odd Functions

(a) Show that the derivative of an odd function is even. That is, if $f(-x) = -f(x)$, then $f'(-x) = f'(x)$.

(b) Show that the derivative of an even function is odd. That is, if $f(-x) = f(x)$, then $f'(-x) = -f'(x)$.

180. Proof Let u be a differentiable function of x. Use the fact that $|u| = \sqrt{u^2}$ to prove that

$$\frac{d}{dx}[|u|] = u'\frac{u}{|u|}, \quad u \neq 0.$$

Using Absolute Value **In Exercises 181–184, use the result of Exercise 180 to find the derivative of the function.**

181. $g(x) = |3x - 5|$ **182.** $f(x) = |x^2 - 9|$

183. $h(x) = |x| \cos x$ **184.** $f(x) = |\sin x|$

True or False? **In Exercises 185 and 186, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

185. If y is a differentiable function of u, and u is a differentiable function of x, then y is a differentiable function of x.

186. If y is a differentiable function of u, u is a differentiable function of v, and v is a differentiable function of x, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}.$$

Linear and Quadratic Approximations **The linear and quadratic approximations of a function f at $x = a$ are**

$P_1(x) = f'(a)(x - a) + f(a)$ **and**

$P_2(x) = \frac{1}{2}f''(a)(x - a)^2 + f'(a)(x - a) + f(a).$

In Exercises 187–190, (a) find the specified linear and quadratic approximations of f, (b) use a graphing utility to graph f and the approximations, (c) determine whether P_1 or P_2 is the better approximation, and (d) state how the accuracy changes as you move farther from $x = a$.

187. $f(x) = \tan x; \quad a = \frac{\pi}{4}$ **188.** $f(x) = \sec x; \quad a = \frac{\pi}{6}$

189. $f(x) = e^x; \quad a = 0$ **190.** $f(x) = \ln x; \quad a = 1$

Calculus AP® – Exam Preparation Questions

191. Multiple Choice Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are shown in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	0	−4	−3	−2
2	−4	−3	−3	3
3	−4	4	5	14
4	6	17	27	31

If $h(x) = xf(x) + g(2x - 5)$, then $h'(3) =$

(A) 4. (B) 6. (C) 12. (D) 36.

192. Multiple Choice If $h(\theta) = \cos^3 8\theta$, then $h'(\theta) =$

(A) $-24 \cos^2 8\theta$. (B) $-3 \cos^2 8\theta \sin 8\theta$.

(C) $-24 \cos^2 8\theta \sin 8\theta$.

(D) $-24 \cos 8\theta \sin 8\theta$.

193. Free Response The function g is defined as $g(x) = (2x^2 + 1)^3$.

(a) Find the slope of the tangent line to the graph of the g at $x = 1$.

(b) Find the equation of the tangent line to the graph of g at $x = 1$.

(c) Determine the point(s), if any, at which the graph of g has a horizontal tangent.

(d) Find $g''(x)$.