Here are some terms which you must know. They are basic to any calculus course.

Critical Values (points): x-values on the function where the function has a slope equal to zero or where the function is not differentiable. Visually, it is where there is a horizontal tangent line or a vertical tangent line.

Stationary Point: A point on the function where there is a horizontal tangent at the x-value.

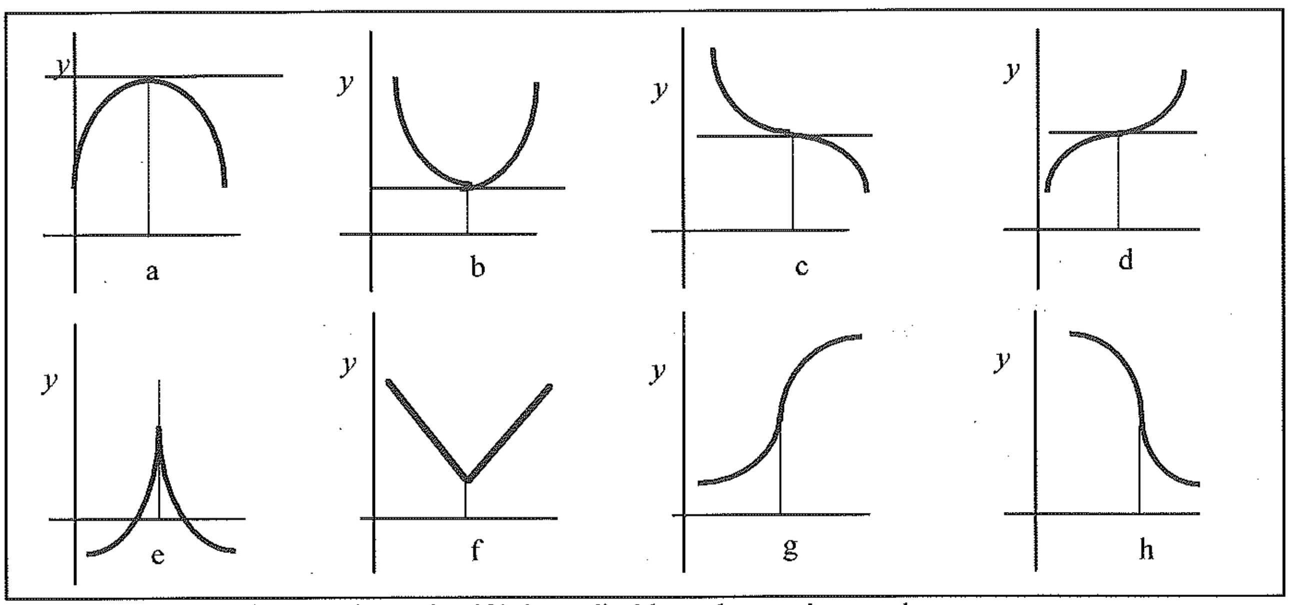
Relative Minimum: Informal definition: the bottom of a hill. Relative minimums occur where the curves switches from decreasing to increasing. The x-value is **where** the relative minimum occurs. The y-value is **what** the relative minimum is. The first derivative test says that a relative minimum occurs when f' switches from negative to positive. The second derivative test say that when $f'(c)=0$ and $f''(c)>0$, there is a relative minimum at $x = c$.

Relative Maximum: Informal definition: the top of a hill. Relative maximums occur where the curve switches from increasing to decreasing. The x-value is **where** the relative maximum occurs. The y-value is **what** the relative maximum is. The first derivative test says that a relative maximum occurs when f' switches from positive to negative. The second derivative test say that when $f'(c)=0$ and $f''(c)<0$, there is a relative maximum at $x = c$.

Relative Extrema: Either a relative minimum or relative maximum.

Absolute maximum or minimum. The highest (lowest) point on the curve. The x-value is **where** the absolute maximum(minimum) occurs. The y-value is **what** the absolute maximum (minimum) is. The absolute minimum or maximum must occur at relative extrema or at the endpoints of an interval.

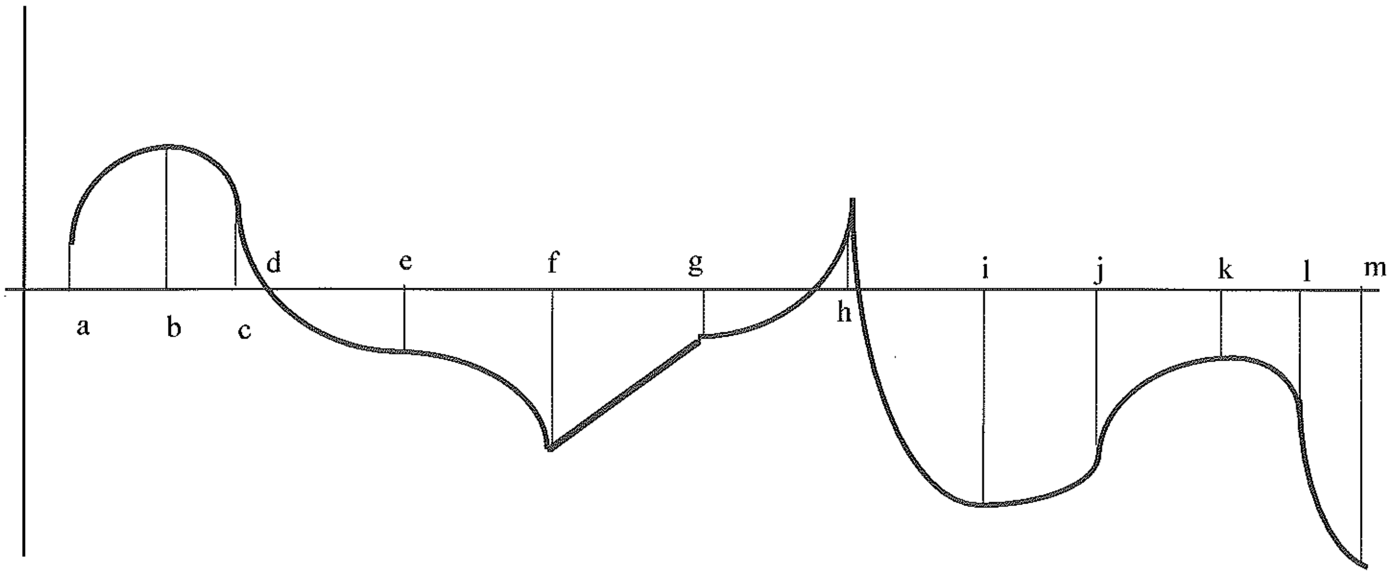
Inflection point: The x-value where the curve switches concavity. Inflection points can occur where the second derivative equals zero or fails to exist. They are found at points where f'' switches sign.



Example 1) For each term, determine if it is applicable at the x-values a - h.

	Critical Point	Relative Minimum	Relative Maximum	Stationary Point	Inflection Point
a					
b					
c					
d					
e					
f					
g					
h					

Function Analysis - Homework

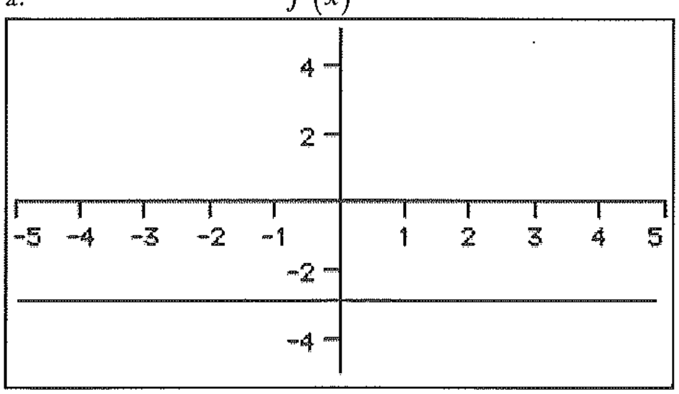


1. For each term, determine if it is applicable at the x-values a - m.

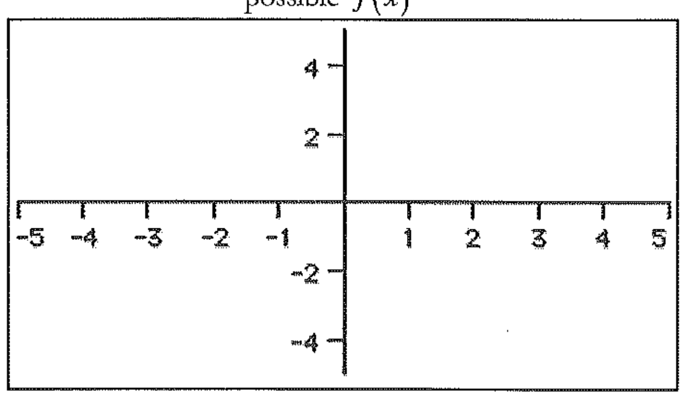
	Critical Point	Relative Minimum	Relative Maximum	Stationary Point	Inflection Point	Absolute Minimum	Absolute Maximum
a							
b							
c							
d							
e							
f							
g							
h							
i							
j							
k							
l							
m							

2) You are given a graph of $f'(x)$. Draw a picture of a possible $f(x)$.

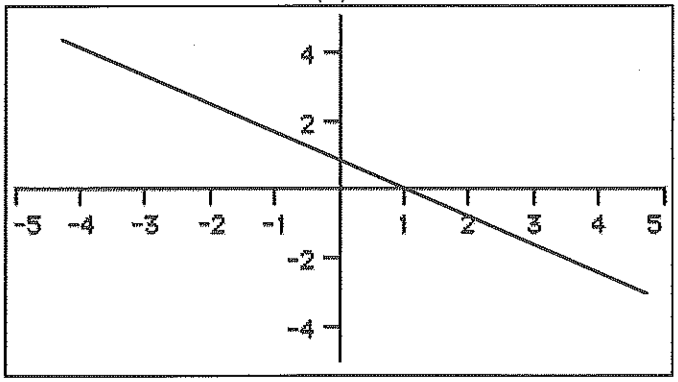
a. $f'(x)$

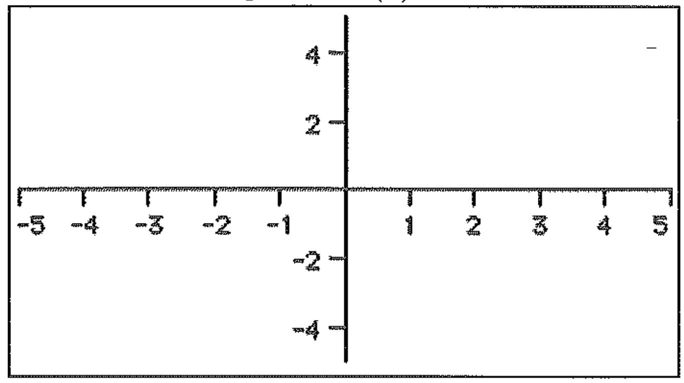


possible $f(x)$

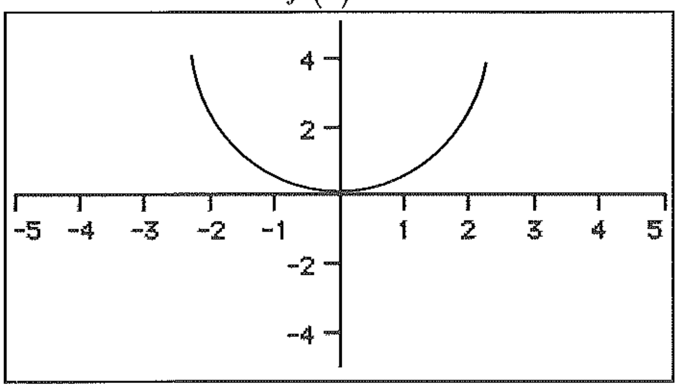


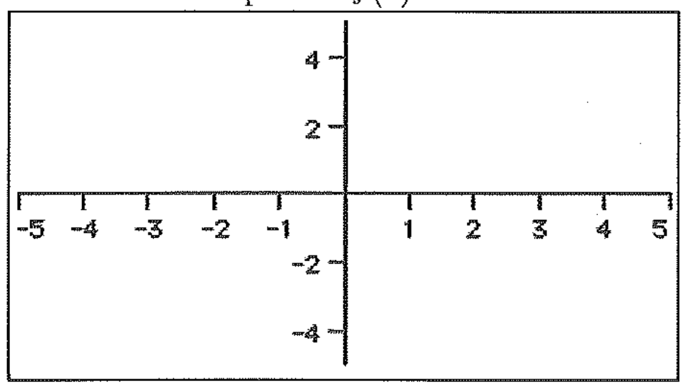
b. $f'(x)$ possible $f(x)$



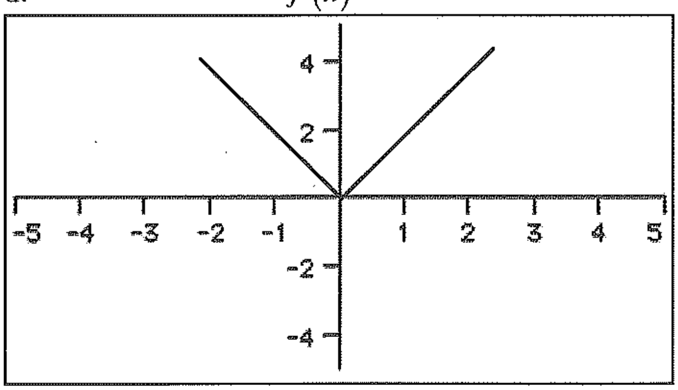


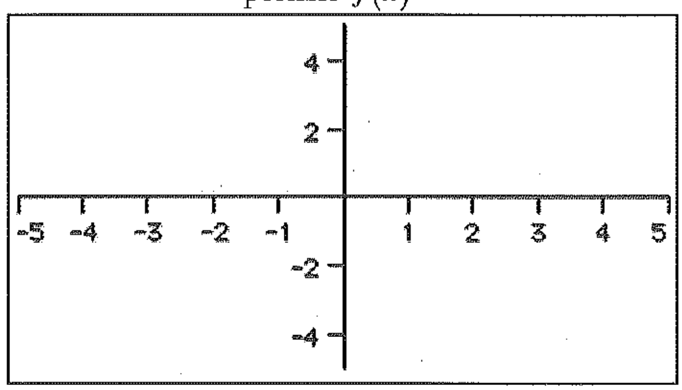
c. $f'(x)$ possible $f(x)$



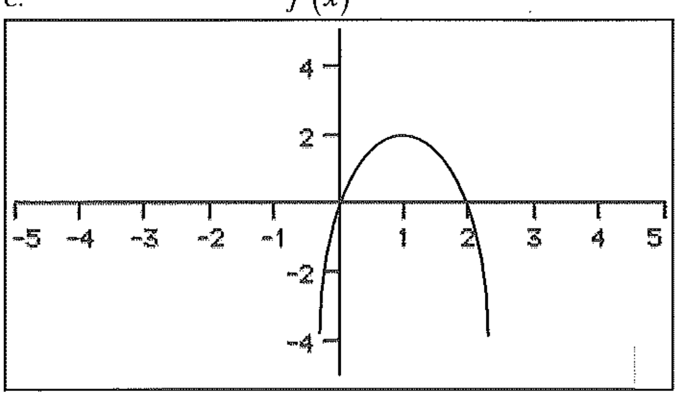


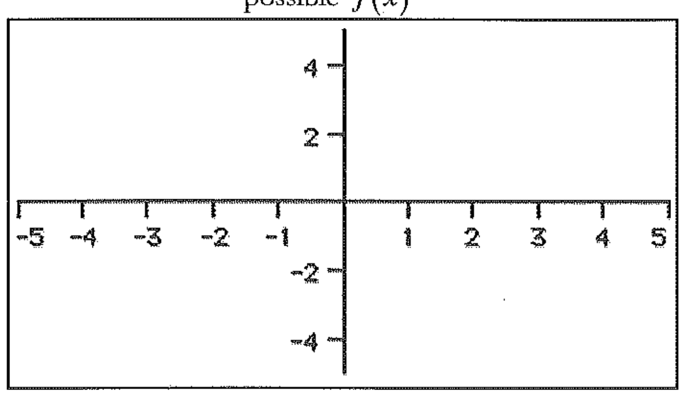
d. $f'(x)$ possible $f(x)$



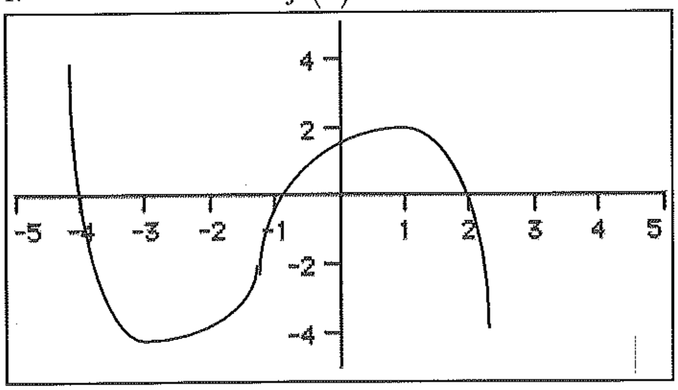


e. $f'(x)$ possible $f(x)$

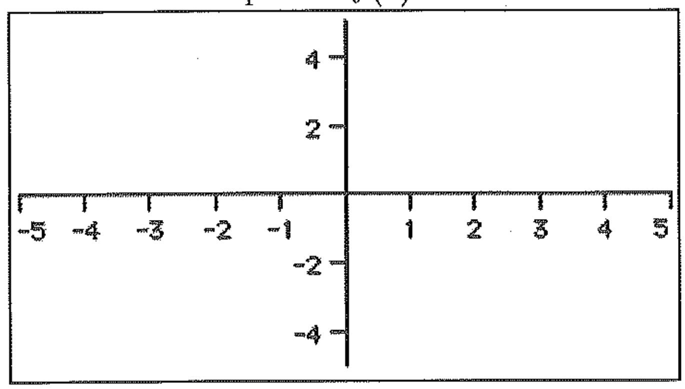




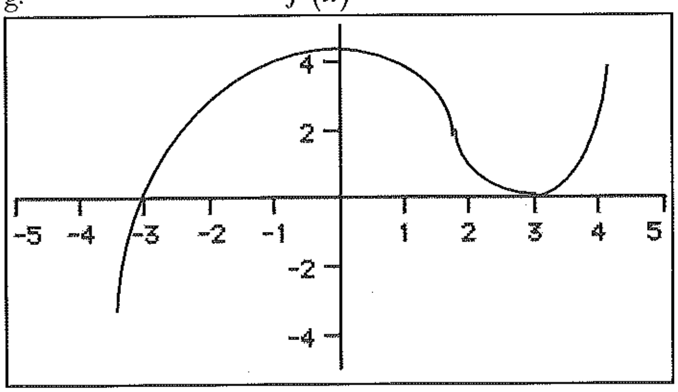
f. $f'(x)$



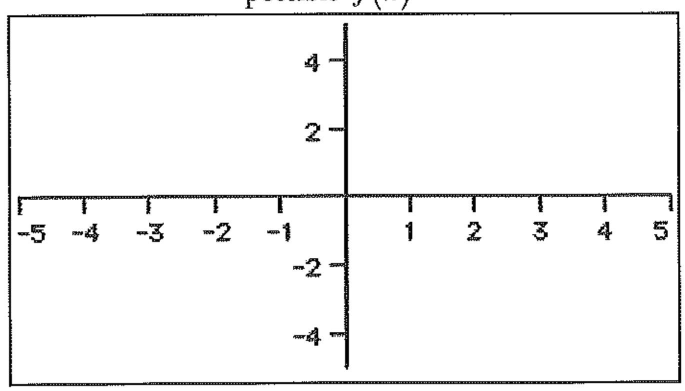
possible $f(x)$



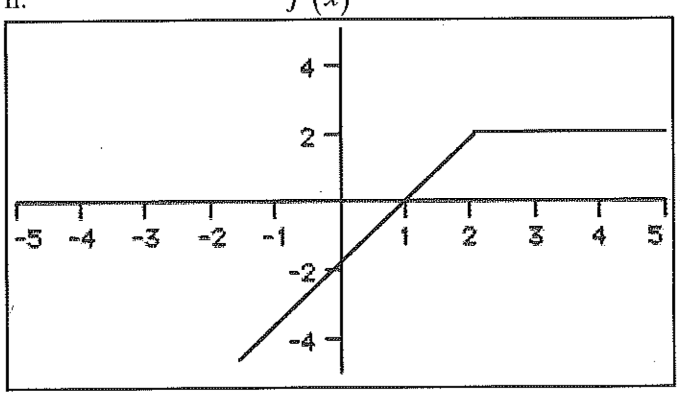
g. $f'(x)$



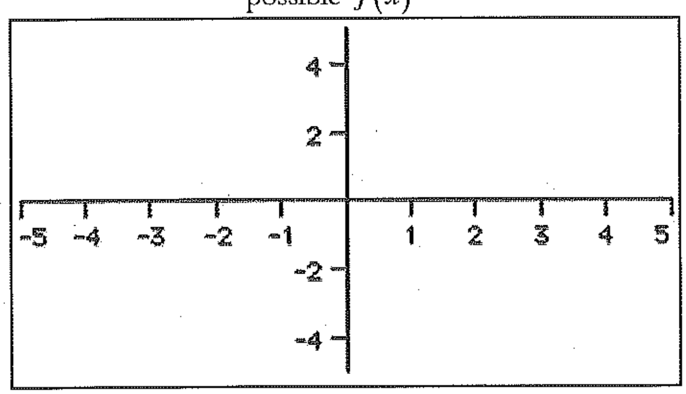
possible $f(x)$



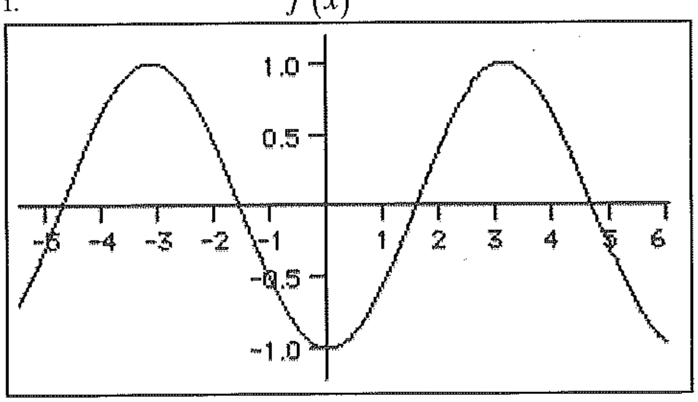
h. $f'(x)$



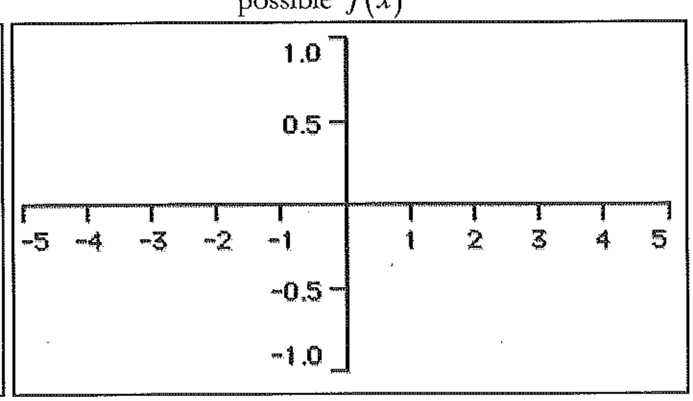
possible $f(x)$



i. $f'(x)$

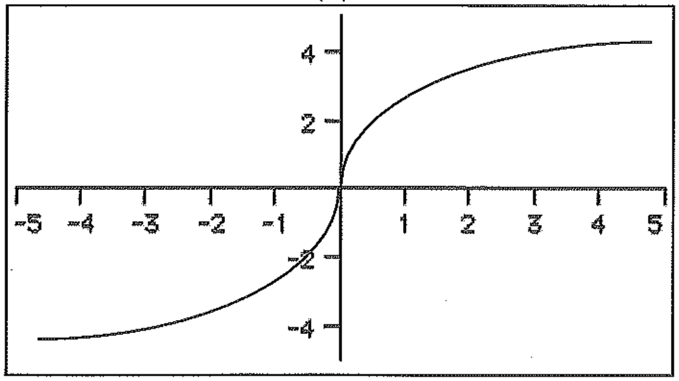


possible $f(x)$

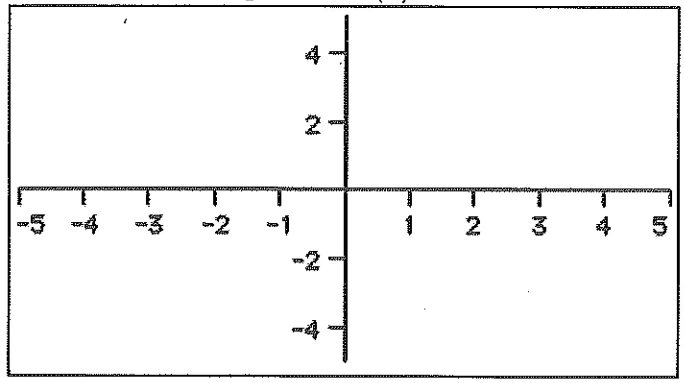


3) You are given a graph of $f'(x)$. Draw a picture of a possible $f(x)$.

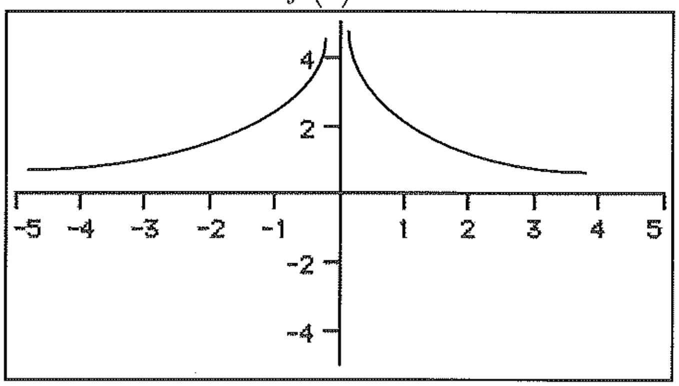
a. $f'(x)$



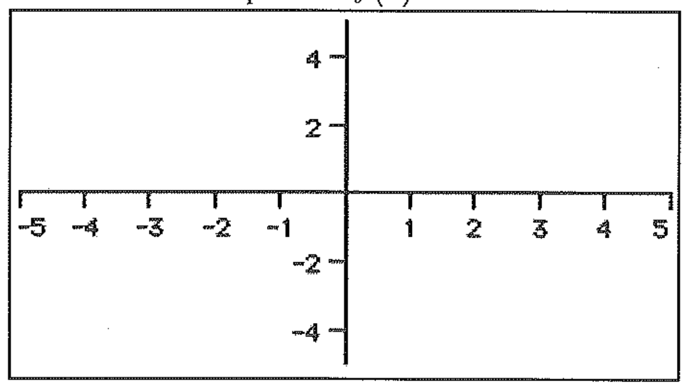
possible $f(x)$



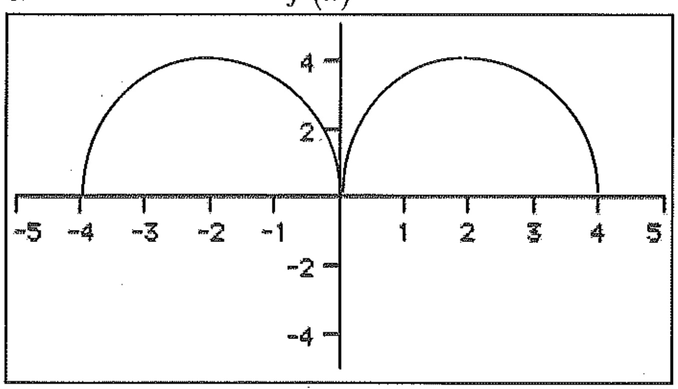
b. $f'(x)$



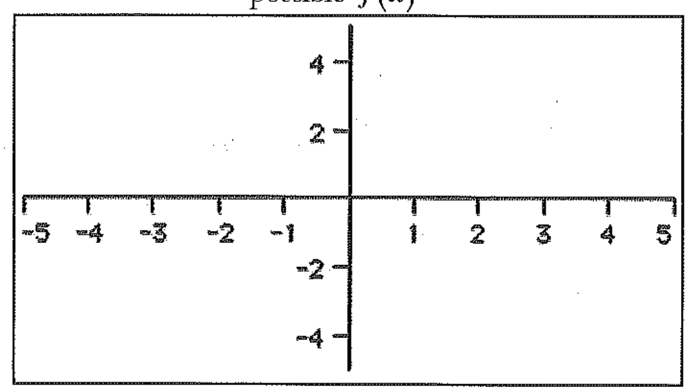
possible $f(x)$



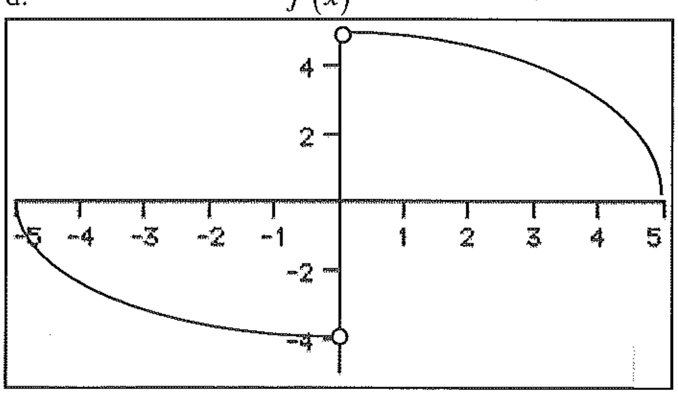
c. $f'(x)$



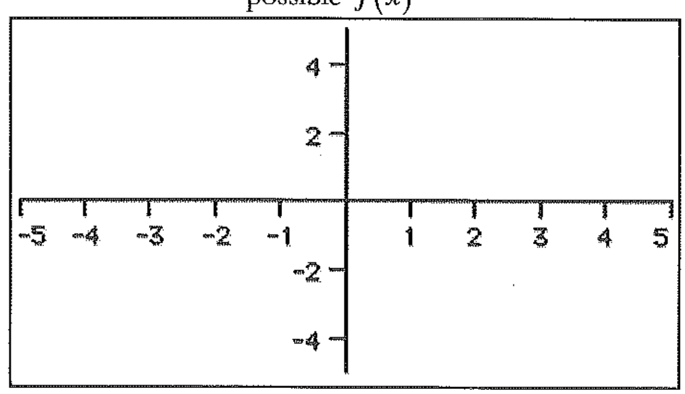
possible $f(x)$



d. $f'(x)$

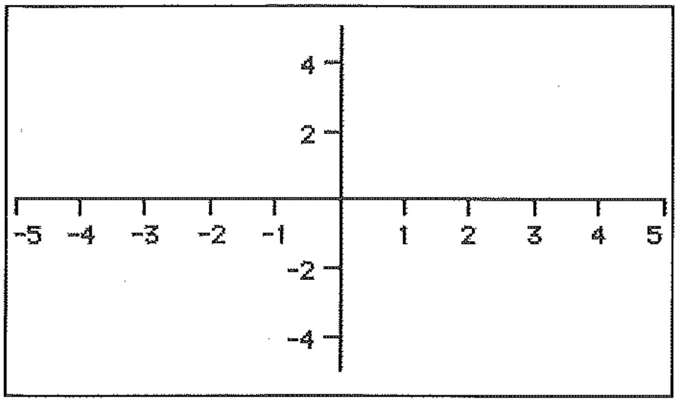


possible $f(x)$

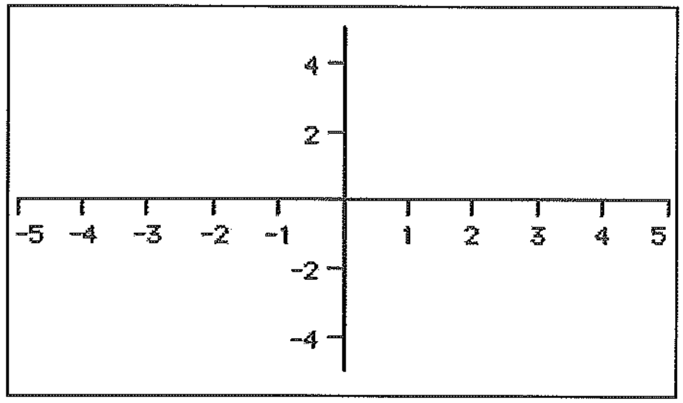


4) Sketch a possible $f(x)$ given the following information.

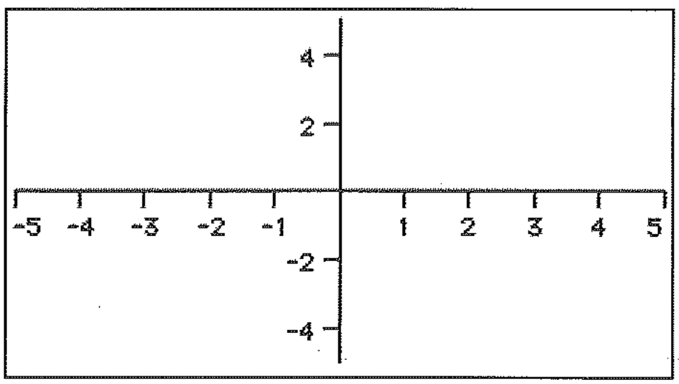
a. $f'(x) > 0, \quad f''(x) < 0$
$f(0) = 2$



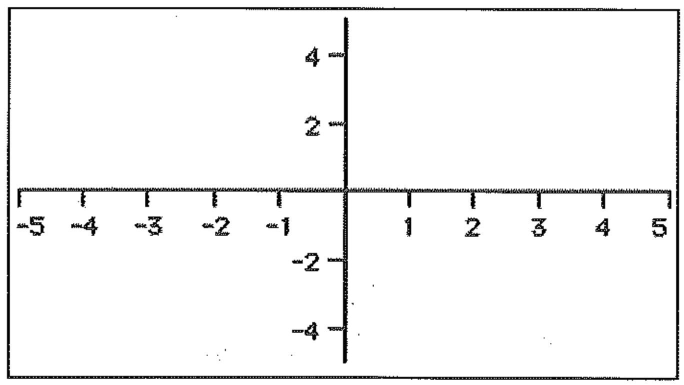
b. $f'(x) > 0, x > 1, \quad f'(x) = -1, x < 1$
$f(1) = -1 \quad \lim_{x\to\infty} f(x) = 4$



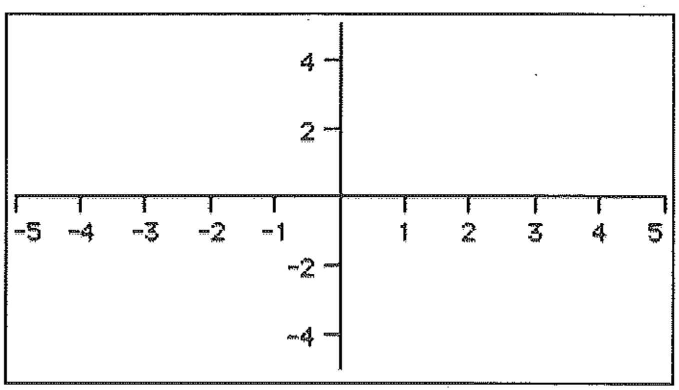
c. $f'(x) < 0, x < 2 \quad f''(x) > 0, x < 2$
$f(x) = 1, x \geq 2 \quad y - \text{intercept} = 2$



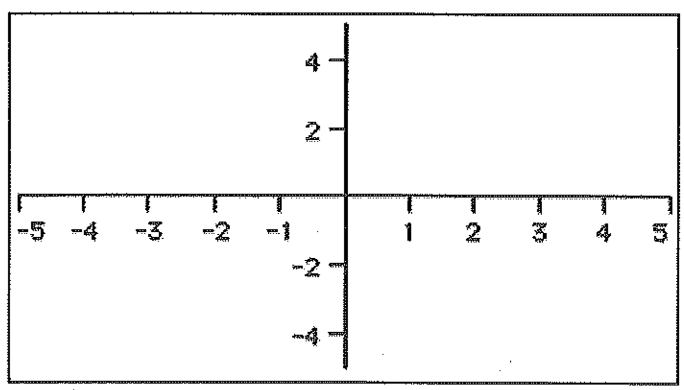
d. $f'(x) < 0, x < 0 \quad f'(x) > 0, x > 0$
$f''(x) > 0 \quad f(0) = -1$



e. $f'(x) > 0 \quad f(0) = 0$
$\lim_{x\to\infty} f(x) = 1 \quad \lim_{x\to-\infty} f(x) = -1$

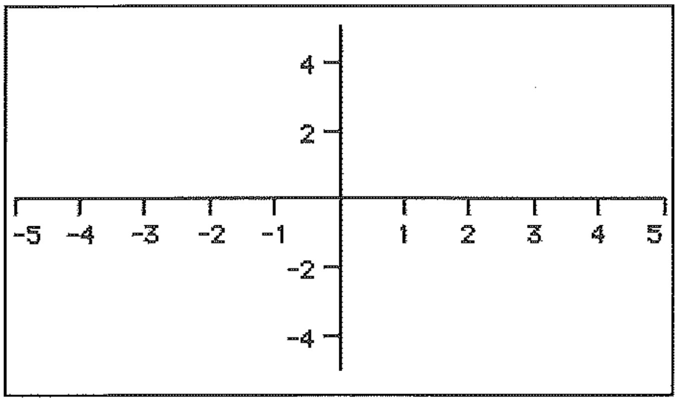


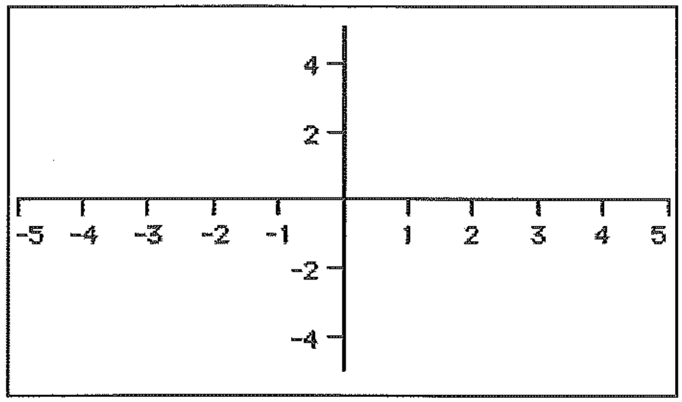
f. $f'(x) > 0, x < 0 \quad f'(x) > 0, x > 3 \quad f'(x) < 0, 0 < x < 3$
$f'(0) = 0 \quad f(0) = 3 \quad f(3) = 0$
$f''(x) < 0, x < 3 \quad f''(x) > 0, x > 3$



g.
$f'(x) < 0, x < 0$ $f'(x) < 0, x > 3$ $f'(x) > 0, 0 < x < 3$
$f'(0) = 0$ $f(0) = -2$ $f''(-2) = 0$
$\lim_{x \to \pm\infty} f(x) = 0$ $\lim_{x \to 3} f(x) = \infty$

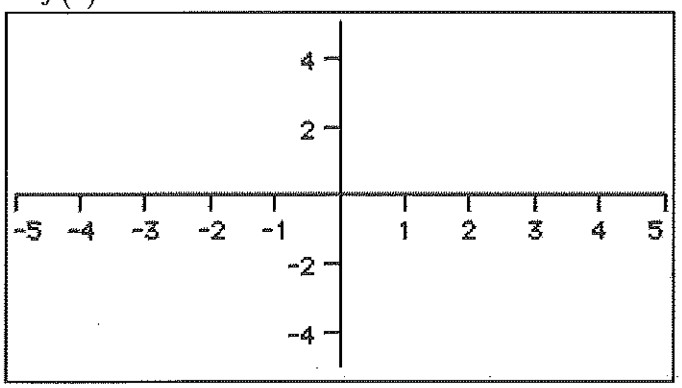
h.
$f'(x) > 0, x \neq 0$ $f'(0) = 0$
$f''(x) < 0, x < 0$ $f''(x) > 0, x > 0$
$f(0) = 1$

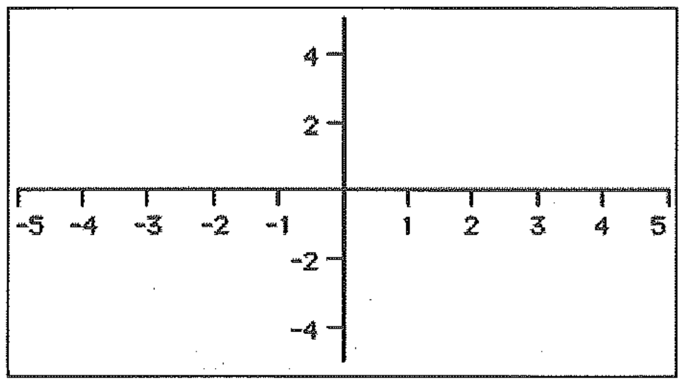




i.
$f'(x) > 0, x \neq 0$ $f'(0)$ *DNE*
$f''(x) > 0, x < 0$ $f''(x) < 0, x > 0$
$f(0) = 1$

j.
$f'(x) < 0, x > 0$ $f''(x) < 0, x > 0$
$\lim_{x \to 0^+} f(x) = 4$ f is symmetric to the origin





5) Find all points of relative maximum and relative minimum and points of inflection if any. Justify your answers. Confirm by calculator.

a. $f(x) = x^2 - 8x + 4$ b. $f(x) = 1 + 12x - 3x^2 - 2x^3$ c. $f(x) = (2x - 5)^3$

d. $f(x) = 3\sqrt[3]{x} - 2$

e. $f(x) = \dfrac{x^2}{x^2 - 4}$ (don't do inflection pts)

f. $f(x) = \sin^2 x + \sin x \quad [0, 2\pi]$
don't do inflection pts)

g. $f(x) = x - \cos x \quad [0, 2\pi]$

h. $f(x) = x\sqrt{x+1}$ (don't do inflection pts)

i. $f(x) = (x^2 - 16)^{2/3}$ (don't do inflection pts)