

## Differentiation of the Natural Log Function - Homework

Find the derivative  $dy/dx$  of the following expressions:

1.  $y = \ln x^6$

2.  $y = \ln(x^2 - 5x - 2)$

3.  $y = (\ln x)^7$

4.  $y = x^3 \ln x$

5.  $y = \frac{x^4}{\ln x}$

6.  $y = \frac{\ln x}{x^4}$

7.  $y = x^2 - 4 \ln x - 2$

8.  $y = \ln(\ln 3x^2)$

9.  $y = \ln \sin x$

10.  $y = \sin(\ln x)$

11.  $y = x \ln \tan x$

12.  $y = \frac{\tan x}{\ln x}$

13.  $y = \ln[(3x^2 - 3x + 2)(5x - 1)]$

14.  $y = \ln \frac{3x^2 - 3x + 2}{5x - 1}$

15.  $y = \ln \sqrt{x^2 - 4x - 7}$

16.  $y = \ln \sqrt[3]{3x^3 - 2x^2 + 5x - 1}$

17.  $y = \ln \sqrt{\frac{2x - 1}{2x + 1}}$

18.  $y = \ln \left[ x \cdot \sqrt[3]{\frac{2x - 1}{2x + 1}} \right]$

## the Natural Log Function and Integration - Classwork

The derivative rules which we just learned will now produce the following integration rules:

$$\int \frac{1}{x} dx = \ln|x| + C \quad \text{and if } u \text{ is a differentiable function of } x, \int \frac{1}{u} du = \ln|u| + C$$

Examples) Find the following:

1)  $\int \frac{4}{x} dx$

2)  $\int \frac{1}{5x-2} dx$

3)  $\int \frac{4}{3-6x} dx$

4)  $\int \frac{7x}{x^2-4} dx$

When you take integrals of fractions, you usually think  $u$ -substitution with the  $u$  being the denominator generating a  $\ln$  function. But not always.

5)  $\int \frac{x}{\sqrt{16-x^2}} dx$

6)  $\int \frac{1}{\sqrt[3]{2x-1}} dx$

7)  $\int \frac{(\ln x)^4}{x} dx$

8)  $\int \frac{x^2-2x+1}{x} dx$

9)  $\int \tan x dx$

10)  $\int \cot 3x dx$

11)  $\int \frac{\cos x}{2 + \sin x} dx$

12)  $\int \frac{1}{\cos^2 x \tan x} dx$

13)  $\int_0^4 \frac{4}{2x+1} dx$

14)  $\int_e^{e^2} \frac{1}{x \ln x} dx$

15)  $\int_0^{\pi/2} \frac{\sin x}{1 + \cos x} dx$

## Derivatives and Integrals of Expressions with “e” - Homework

Find the derivatives of the following functions:

1.  $y = e^{4x}$

2.  $y = 16e^{-2x}$

3.  $y = x^3e^x$

4.  $y = \frac{e^x}{x^6}$

5.  $y = e^x \tan x$

6.  $y = 2 \operatorname{cose}^x$

7.  $y = \frac{e^x}{\ln x}$

8.  $y = \frac{\ln x}{e^x}$

9.  $y = (e^x - 2x - 1)^3$

10.  $y = \sqrt{e^{3x} - 4x}$

11.  $y = -4e^{\operatorname{csc} x}$

12.  $y = \frac{3}{e^x + e^{-x}}$

13.  $y = \ln\left(\frac{1 - e^x}{1 + e^x}\right)$

14.  $y = e^x(\sin x - \cos x)$

15.  $y = \pi^x$

16.  $y = 10^{x^2 - \sin x}$

Use implicit differentiation to find  $dy/dx$

17.  $xe^y - 2x - 3y = 0$

Find the second derivative of the following

18.  $y = \sqrt{x} - e^x \ln x$

Find the extrema for the following functions:

19.  $y = e^{\left(\frac{-x^2}{2}\right)}$

20.  $y = 2xe^{-x}$

21.  $y = x^2e^{-x}$

Find the following integrals

22.  $\int e^{6x} dx$

23.  $\int 4e^{-2x} dx$

24.  $\int \sin x \cdot e^{\cos x} dx$

25.  $\int \frac{e^{\tan x}}{\cos^2 x} dx$

26.  $\int (e^x + e^{-x})^2 dx$

27.  $\int e^x \sqrt{4 - e^x} dx$

28.  $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

29.  $\int \frac{e^{2x} + 2e^{-x} + 1}{e^x} dx$

30.  $\int \frac{e^{-x}}{4 + e^{-x}} dx$

31.  $\int e^{-x} \tan(e^{-x}) dx$

32.  $\int_1^9 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$

33.  $\int_1^{e^3} \frac{1}{x} dx$

## Differentiation of the Natural Log Function - Homework

Find the derivative  $dy/dx$  of the following expressions:

1.  $y = \ln x^6$

$$y' = \frac{1}{x^6} \cdot 6x^5 = \frac{6}{x}$$

2.  $y = \ln(x^2 - 5x - 2)$

3.  $y = (\ln x)^7$

$$y' = 7(\ln x)^6 \cdot \frac{1}{x}$$

4.  $y = x^3 \ln x$

PROD

5.  $y = \frac{x^4}{\ln x}$

QUO

6.  $y = \frac{\ln x}{x^4}$

$$y' = \frac{x^4 \cdot \frac{1}{x} - \ln x \cdot 4x^3}{x^8} = \frac{1 - 4\ln x}{x^5}$$

7.  $y = x^2 - 4 \ln x - 2$

8.  $y = \ln(\ln 3x^2)$

$$y' = \frac{1}{\ln^2 3x^2} \cdot \frac{1}{3x^2} \cdot 6x$$

9.  $y = \ln \sin x$

CHAIN

10.  $y = \sin(\ln x)$

$$y' = \frac{\cos(\ln x)}{x}$$

11.  $y = x \ln \tan x$

PROD, CHAIN

12.  $y = \frac{\tan x}{\ln x}$

QUO

13.  $y = \ln[(3x^2 - 3x + 2)(5x - 1)]$

CHAIN, PROD.

14.  $y = \ln \frac{3x^2 - 3x + 2}{5x - 1}$

$$y' = \frac{(5x-1)}{3x^2-3x+2} \cdot \frac{(5x-1)(6x-3) - (3x^2-3x+2) \cdot 5}{(5x-1)^2}$$

15.  $y = \ln \sqrt{x^2 - 4x - 7}$

16.  $y = \ln \sqrt[5]{3x^3 - 2x^2 + 5x - 1}$

17.  $y = \ln \sqrt{\frac{2x-1}{2x+1}}$

18.  $y = \ln \left[ x \cdot \sqrt[3]{\frac{2x-1}{2x+1}} \right]$

$$y' = \frac{1}{(3x^3 - 2x^2 + 5x - 1)^{4/5}} \cdot (3x^3 - 2x^2 + 5x - 1)^{-4/5} \cdot (9x^2 - 4x + 5)$$

$$= \frac{9x^2 - 4x + 5}{3x^3 - 2x^2 + 5x - 1}$$

## the Natural Log Function and Integration - Classwork

The derivative rules which we just learned will now produce the following integration rules:

$$\int \frac{1}{x} dx = \ln|x| + C \quad \text{and if } u \text{ is a differentiable function of } x, \int \frac{1}{u} du = \ln|u| + C$$

Examples) Find the following:

1)  $\int \frac{4}{x} dx$

$$= 4 \ln|x| + C.$$

2)  $\int \frac{1}{5x-2} dx$

3)  $\int \frac{4}{3-6x} dx$

$$\begin{aligned} u &= 3-6x & \int \frac{4}{6} \frac{1}{u} du \\ du &= -6 dx & = \frac{4}{6} \ln|u| + C \\ \frac{-1}{6} du &= dx & = -\frac{2}{3} \ln|3-6x| + C. \end{aligned}$$

4)  $\int \frac{7x}{x^2-4} dx$

$$u = x^2 - 4$$

When you take integrals of fractions, you usually think  $u$ -substitution with the  $u$  being the denominator generating a  $\ln$  function. But not always.

5)  $\int \frac{x}{\sqrt{16-x^2}} dx$

$$\begin{aligned} u &= 16-x^2 & \Rightarrow \int -\frac{1}{2} u^{-1/2} du \\ du &= -2x dx & = -u^{1/2} + C \\ -\frac{1}{2} du &= x dx & = -\sqrt{16-x^2} + C. \end{aligned}$$

6)  $\int \frac{1}{\sqrt[3]{2x-1}} dx$

7)  $\int \frac{(\ln x)^4}{x} dx$

$$\begin{aligned} u &= \ln x \\ du &= \frac{1}{x} dx \\ \int u^4 du &= \frac{1}{5} u^5 + C \\ &= \frac{1}{5} (\ln x)^5 + C. \end{aligned}$$

8)  $\int \frac{x^2-2x+1}{x} dx$

Divide!

9)  $\int \tan x dx$

$$= \int \frac{\sin x}{\cos x} dx$$

$$u = \cos x \dots$$

10)  $\int \cot 3x dx$

11)  $\int \frac{\cos x}{2 + \sin x} dx$

12)  $\int \frac{1}{\cos^2 x \tan x} dx$

$$= \int \frac{1}{\tan x} \sec^2 x dx$$

$$u = \tan x \dots$$

13)  $\int_0^4 \frac{4}{2x+1} dx$

14)  $\int_e^{e^2} \frac{1}{x \ln x} dx$

$$\begin{aligned} u &= \ln x \\ du &= \frac{1}{x} dx \end{aligned}$$

15)  $\int_0^{\pi/2} \frac{\sin x}{1 + \cos x} dx$

$$u = 1 + \cos x$$

## Derivatives and Integrals of Expressions with "e" - Homework

Find the derivatives of the following functions:

1.  $y = e^{4x}$

$$y' = 4e^{4x}$$

2.  $y = 16e^{-2x}$

3.  $y = x^3 e^x$

$$y' = 3x^2 e^x + x^3 e^x$$

4.  $y = \frac{e^x}{x^6} = e^x \cdot x^{-6}$   
PROD.

5.  $y = e^x \tan x$

6.  $y = 2 \cos e^x$

$$y' = -2 \sin e^x \cdot e^x$$

7.  $y = \frac{e^x}{\ln x}$   
HI / LO

$$y' = \frac{\ln x \cdot e^x - \frac{e^x}{1}}{(\ln x)^2}$$

8.  $y = \frac{\ln x}{e^x}$

9.  $y = (e^x - 2x - 1)^3$

10.  $y = \sqrt{e^{3x} - 4x}$

11.  $y = -4e^{\sec x}$   
FNNECR

12.  $y = \frac{3}{e^x + e^{-x}} = 3(e^x + e^{-x})^{-1}$

$$y' = -4e^{\sec x} \cdot \sec x \tan x$$

13.  $y = \ln\left(\frac{1-e^x}{1+e^x}\right)$

14.  $y = e^x(\sin x - \cos x)$

15.  $y = \pi^x$

16.  $y = 10^{x^2 - \sin x}$

$$y' = \left(\frac{1+e^x}{1-e^x}\right) \cdot \frac{(1+e^x)(-e^x) - (1-e^x)e^x}{(1+e^x)^2}$$

Use implicit differentiation to find  $dy/dx$

17.  $xe^y - 2x - 3y = 0$

$$e^y \frac{dy}{dx} + xe^y - 2 - 3 \frac{dy}{dx} = 0$$

$$(e^y - 3) \frac{dy}{dx} = -xe^y + 2$$

$$\frac{dy}{dx} = \frac{-xe^y + 2}{e^y - 3}$$

Find the second derivative of the following

18.  $y = \sqrt{x} - e^x \ln x$

$$y' = x^{1/2} - (e^x \ln x + e^x \cdot \frac{1}{x})$$

$$y'' = \frac{1}{2}x^{-1/2} - (e^x \ln x + \frac{e^x}{x}) - \frac{xe^x - e^x}{x^2}$$

Find the extrema for the following functions:

$$19. y = e^{\left(\frac{-x^2}{2}\right)}$$

$$y' = e^{-x^2/2} \cdot (-x)$$

$$0 = -e^{-x^2/2} \cdot x$$

$$x = 0.$$

$$20. y = 2xe^{-x}$$

$$y' = 2e^{-x} + 2x e^{-x}(-1)$$

$$0 = \frac{2e^{-x} - 2x e^{-x}}{2e^{-x}}$$

$$x = 1.$$

$$21. y = x^2 e^{-x}$$

Find the following integrals

$$22. \int e^{6x} dx$$

$$u = 6x.$$

$$23. \int 4e^{-2x} dx$$

$$24. \int \sin x \cdot e^{\cos x} dx$$

$$u = \cos x$$

$$25. \int \frac{e^{\tan x}}{\cos^2 x} dx = \int e^{\tan x} \cdot \sec^2 x dx$$

$$26. \int (e^x + e^{-x})^2 dx$$

$$\text{EXPAND,}$$

$$27. \int e^x \sqrt{4 - e^x} dx$$

$$u = 4 - e^x$$

$$28. \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$$

$$u = e^x - e^{-x}$$

$$du = e^x + e^{-x}$$

$$29. \int \frac{e^{2x} + 2e^{-x} + 1}{e^x} dx$$

$$\text{DIVIDE.}$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$= \ln|e^x - e^{-x}| + C.$$

$$30. \int \frac{e^{-x}}{4 + e^{-x}} dx$$

$$u = 4 + e^{-x}$$

$$31. \int e^{-x} \tan(e^{-x}) dx$$

$$u = e^{-x}$$

SEE #9 ON  
SECOND PAGE!

$$32. \int_1^9 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

$$= \int_1^9 e^{\sqrt{x}} \cdot \frac{1}{2} e^{-1/2} dx$$

$$33. \int_1^{e^3} \frac{1}{x} dx = \ln|x| \Big|_1^{e^3}$$

$$= \ln e^3 - \ln 1$$

$$= 3 - 0$$

$$= \underline{\underline{3}}$$