Definite Integration with u-Substitution - Classwork

When you have to find a definite integral involving *u-substitution*, it is often convenient to determine the limits of integration in terms of the variable u, rather than having to integrate and switch back to x and then substitute. This is called "changing the limits."

Example 1) $\int_0^2 x\left(x^2+1\right)^2 dx$ Start off by finding u ________ $x=2, u=$ ________

$\frac{1}{2}\int_0^2 2x\left(x^2+1\right)^2 dx$ $du=$ ________ Do we have it? $x=0, u=$ ________

Now write everything in terms of u and calculate. It is no longer necessary to switch back to x.

Example 2) $\int_0^1 x\sqrt{1-x^2}\,dx$

Example 3) $\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}\,dx$

Example 4) $\int_0^{\pi/4} \sin 2x\,dx$

Example 5) $\int_0^{\pi/2} \sin x\sqrt{\cos x}\,dx$

Example 6) $\int_0^{\pi/6} \left(1+\sin x\cos x\right) dx$

Example 7) $\int_{-2}^{2} \left|1-x^2\right| dx$

Definite Integration with u-Substitution - Homework

Find the values of the following definite integrals. Verify using your calculator. Some will use *u*-substitution, others will not.

1. $\int_{-2}^{2} \left(x^3 - 1\right)\, dx$

2. $\int_{0}^{4} x\left(\sqrt{x} - 1\right)\, dx$

3. $\int_{0}^{\pi/3} \sin(2x)\, dx$

4. $\int_{0}^{\pi/12} \left(1 - \cos 2x\right)\, dx$

5. $\int_{0}^{1} 2x\left(x^2 + 1\right)^2\, dx$

6. $\int_{0}^{3} x\sqrt{9 - x^2}\, dx$

7. $\int_{0}^{5} \left|x - 4\right|\, dx$

8. $\int_{0}^{4} \left|x - \sqrt{x}\right|\, dx$

9. $\int_{2}^{3} \frac{x}{\left(x^2 - 3\right)^2}\, dx$

10. $\int_{0}^{4} \frac{dt}{\sqrt{2t + 1}}$

11. $\int_{0}^{\pi/2} \cos^3 t \sin t\, dt$

12. $\int_{0}^{\sqrt{\pi/2}} t \sin\left(\pi - t^2\right)\, dt$

13. $\int_{0}^{\pi/4} \sqrt{\tan x}\, \sec^2 x\, dx$

14. $\int_{0}^{\pi/3} \cos x \sqrt{1-\cos^2 x}\, dx$

15. $\int_{0}^{1} x\sqrt{ax^2+b}\, dx$

16. $\int_{\pi^2/4}^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

17. $\int_{0}^{4} \left|9-x^2\right| dx$

18. $\int_{-4}^{4} \frac{1}{x^2}\, dx$ (Be careful!)

If $\int_{0}^{2} f(x)\, dx = \frac{11}{3}$ and $\int_{0}^{6} f(x)\, dx = 15$, $f(x)$ is an even function (symmetric to the y - axis), find the following:

19. $\int_{-2}^{0} f(x)\, dx$ 20. $\int_{-2}^{2} f(x)\, dx$ 21. $\int_{0}^{2} -f(x)\, dx$ 22. $\int_{-2}^{0} 3f(x)\, dx$ 23. $\int_{0}^{2} f(3x)\, dx$

If $\int_{0}^{2} f(x)\, dx = \frac{11}{3}$ and $\int_{0}^{6} f(x)\, dx = 15$, $f(x)$ is an odd function (symmetric to the origin), find the following:

24. $\int_{-2}^{0} f(x)\, dx$ 25. $\int_{-2}^{2} f(x)\, dx$ 26. $\int_{0}^{2} -f(x)\, dx$ 27. $\int_{-2}^{0} 3f(x)\, dx$ 28. $\int_{-2}^{2} f(3x)\, dx$