Check Point 5 Rewrite $y = 4(7.8)^x$ in terms of base e. Express the answer in terms of a natural logarithm, and then round to three decimal places.

When using a graphing utility to model data, begin with a scatter plot, drawn either by hand or with the graphing utility, to obtain a general picture for the shape of the data. It might be difficult to determine which model best fits the data—linear, logarithmic, exponential, quadratic, or something else. If necessary, use your graphing utility to fit several models to the data. The best model is the one that yields the value r, the correlation coefficient, closest to 1 or -1. Finding a proper fit for data can be almost as much art as it is mathematics. In this era of technology, the process of creating models that best fit data is one that involves more decision making than computation.

EXERCISE SET 3.5

Practice and Application Exercises

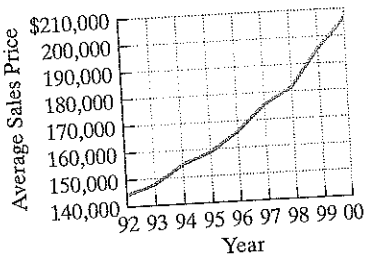
The exponential growth model $A = 203e^{0.011t}$ describes the population of the United States, A, in millions, t years after 1970. Use this model to solve Exercises 1–4.

1. What was the population of the United States in 1970?
2. By what percentage is the population of the United States increasing each year?
3. When will the U.S. population be 300 million?
4. When will the U.S. population be 350 million?

India is currently one of the world's fastest-growing countries. By 2040, the population of India will be larger than the population of China; by 2050, nearly one-third of the world's population will live in these two countries alone. The exponential growth model $A = 574e^{0.026t}$ describes the population of India, A, in millions, t years after 1974. Use this model to solve Exercises 5–8.

5. By what percentage is the population of India increasing each year?
6. What was the population of India in 1974?
7. When will India's population be 1624 million?
8. When will India's population be 2732 million?
9. Low interest rates, easy credit, and strong demand from new immigrants have driven up the average sales price of new one-family houses in the United States. In 1995, the average sales price was $158,700 and by 2000 it had increased to $207,200.
   a. Use the exponential growth model $A = A_0e^{kt}$, in which t is the number of years after 1995, to find the exponential growth function that models the data.
   b. According to your model, by which year will the average sales price of a new one-family house reach $300,000?

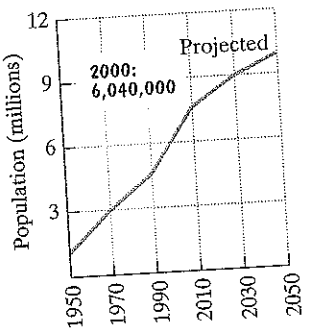
Average Sales Prices of New One-Family Houses



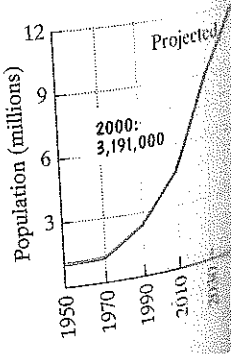
Source: U.S. Census Bureau

About the size of New Jersey, Israel has seen its population soar to more than 6 million since it was established. With the help of U.S. aid, the country now has a diversified economy rivaling those of other developed Western nations. By contrast, the Palestinians, living under Israeli occupation and a corrupt regime, endure bleak conditions. The graphs show that by 2050, Palestinians in the West Bank, Gaza Strip, and East Jerusalem will outnumber Israelis. Exercises 10–12 involve the projected growth of these two populations.

Population of Israel



Palestinian Population in West Bank, Gaza, and East Jerusalem



Source: *Newsweek*

10. In 2000, the population of the Palestinians in the West Bank, Gaza Strip, and East Jerusalem was approximately 3.2 million and by 2050 it is projected to grow to 12 million. Use the exponential growth model $A = A_0e^{kt}$, in which t is the number of years after 2000, to find the exponential growth function that models the data.

11. In 2000, the population of Israel was approximately 6.04 million and by 2050 it is projected to grow to 10 million. Use the exponential growth model $A = A_0e^{kt}$, in which t is the number of years after 2000, to find an exponential growth function that models the data.

12. Use the growth models in Exercises 10 and 11 to determine the year in which the two populations will be the same.

An artifact originally had 16 grams of carbon-14 present. The decay model $A = 16e^{-0.000121t}$ describes the amount of carbon-14 present, A, in grams, after t years. Use this model to solve Exercises 13–14.

13. How many grams of carbon-14 will be present after 5715 years?

14. How many grams of carbon-14 will be present after 11,430 years?

15. The half-life of the radioactive element krypton-91 is 10 seconds. If 16 grams of krypton-91 are initially present, how many grams are present after 10 seconds? 20 seconds? 30 seconds? 40 seconds? 50 seconds?

16. The half-life of the radioactive element plutonium-239 is 25,000 years. If 16 grams of plutonium-239 are initially present how many grams are present after 25,000 years? 50,000 years? 75,000 years? 100,000 years? 125,000 years?

Use the exponential decay model for carbon-14, $A = A_0e^{-0.000121t}$, to solve Exercises 17–18.

17. Prehistoric cave paintings were discovered in a cave in France. The paint contained 15% of the original carbon-14. Estimate the age of the paintings.

18. Skeletons were found at a construction site in San Francisco in 1989. The skeletons contained 88% of the expected amount of carbon-14 found in a living person. In 1989, how old were the skeletons?

19. The August 1978 issue of *National Geographic* described the 1964 find of dinosaur bones of a newly discovered dinosaur weighing 170 pounds, measuring 9 feet, with a 6-inch claw on one toe of each hind foot. The age of the dinosaur was estimated using potassium-40 dating of rocks surrounding the bones.
    a. Potassium-40 decays exponentially with a half-life of approximately 1.31 billion years. Use the fact that after 1.31 billion years a given amount of potassium-40 will have decayed to half the original amount to show that the decay model for potassium-40 is given by $A = A_0e^{-0.52912t}$, where t is in billions of years.
    b. Analysis of the rocks surrounding the dinosaur bones indicated that 94.5% of the original amount of potassium-40 was still present. Let $A = 0.945A_0$ in the model in part (a) and estimate the age of the bones of the dinosaur.

20. A bird species in danger of extinction has a population that is decreasing exponentially ($A = A_0e^{kt}$). Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation will be irreversible. When will this happen?

21. Use the exponential growth model, $A = A_0e^{kt}$, to show that the time it takes a population to double (to grow from A_0 to $2A_0$) is given by $t = \dfrac{\ln 2}{k}$.

22. Use the exponential growth model, $A = A_0e^{kt}$, to show that the time it takes a population to triple (to grow from A_0 to $3A_0$) is given by $t = \dfrac{\ln 3}{k}$.

Use the formula $t = \dfrac{\ln 2}{k}$ that gives the time for a population with a growth rate k to double to solve Exercises 23–24. Express each answer to the nearest whole year.

23. China is growing at a rate of 1.1% per year. How long will it take China to double its population?

24. Japan is growing at a rate of 0.3% per year. How long will it take Japan to double its population?

25. The logistic growth function
$$f(t) = \frac{100{,}000}{1 + 5000e^{-t}}$$
describes the number of people, $f(t)$, who have become ill with influenza t weeks after its initial outbreak in a particular community.
    a. How many people became ill with the flu when the epidemic began?
    b. How many people were ill by the end of the fourth week?
    c. What is the limiting size of the population that becomes ill?

26. The logistic growth function
$$f(t) = \frac{500}{1 + 83.3e^{-0.162t}}$$
describes the population, $f(t)$, of an endangered species of birds t years after they are introduced to a nonthreatening habitat.
    a. How many birds were initially introduced to the habitat?
    b. How many birds are expected in the habitat after 10 years?
    c. What is the limiting size of the bird population that the habitat will sustain?

The logistic growth function
$$P(x) = \frac{90}{1 + 271e^{-0.122x}}$$
models the percentage, $P(x)$, of Americans who are x years